18.06 **Hints and Answers to Problem Set 6**

1. In this problem, the normal equations are $A^TA\mathbf{x} = A^T\mathbf{b}$, where A and b are defined for each problem in the matlab code below. The solution vector $\mathbf{x}$ is the vector of coefficients of the polynomial, so $\mathbf{x} = (C, D, E)^T$ for the quadradic, and $\mathbf{x} = (C, D, E, F)^T$ for the cubic case. Note that fitting a cubic through 4 points is an exactly soluble problem, and instead of solving the normal equations, we can just solve $A\mathbf{x} = \mathbf{b}$.

```
>> diary on
>> A=[1 1 1; 1 1.5 1.5^2; 1 2 2^2; 1 3 3^2]
A =
    1.0000    1.0000    1.0000
    1.0000    1.5000    2.2500
    1.0000    2.0000    4.0000
    1.0000    3.0000    9.0000
>> b=[0; 2; 3; 3]
b =
     0
     2
     3
     3
>> ATA = A'*A
ATA =
    4.0000    7.5000   16.2500
    7.5000   16.2500   39.3750
   16.2500   39.3750  103.0625
>> ATb = A'*b
ATb =
    8.0000
   18.0000
   43.5000
>> ATA\ATb
ans =
   -6.0909
    7.6636
   -1.5455
>>
>> A=[1 1 1 1; 1 1.5 1.5^2 1.5^3; 1 2 2^2 2^3; 1 3 3^2 3^3]
A =
    1.0000    1.0000    1.0000    1.0000
    1.0000    1.5000    2.2500    3.3750
    1.0000    2.0000    4.0000    8.0000
    1.0000    3.0000    9.0000   27.0000
```

```
>> ATA = A'*A
ATA =
    4.0000    7.5000   16.2500   39.3750
    7.5000   16.2500   39.3750  103.0625
   16.2500   39.3750  103.0625  283.5938
   39.3750  103.0625  283.5938  805.3906
>> ATb = A'*b
ATb =
    8.0000
   18.0000
   43.5000
  111.7500
>> ATA\ATb
ans =
   -8.0000
   11.1667
   -3.5000
    0.3333
>> A\b
ans =
   -8.0000
   11.1667
   -3.5000
    0.3333
>> diary off
```

3. In this problem we need to use least squares for two problems. The equation of a line with arbitrary y-intersect is given by $y = C + Dx$, and we need to fit the parameters C and D. For the line through the origin, the equation is $y = Dx$, and we only fit the parameter D.

   The first case fits parameters C and D:

```
>> A = [1 5; 1 10; 1 15; 1 20; 1 25; 1 30; 1 40; 1 50]
A =
     1     5
     1    10
     1    15
     1    20
     1    25
     1    30
     1    40
     1    50
```

```
>> b = [9; 10; 14; 18; 22; 24; 29; 29]
b =
     9
    10
    14
    18
    22
    24
    29
    29
>> ATA = A'*A
ATA =
           8         195
         195        6375
>> ATb = A'*b
ATb =
         155
        4595
>> c1 = ATA\ATb
c1 =
    7.0983
    0.5037
>> e1 = b-(A*c1)
e1 =
   -0.6166
   -2.1349
   -0.6532
    0.8285
    2.3102
    1.7919
    1.7553
   -3.2813
```

The next case fits only parameter D to get a line through the origin:

```
>>
>> A = [5; 10; 15; 20; 25; 30; 40; 50]
A =
     5
    10
    15
    20
    25
    30
    40
    50
```

```
>> ATA = A'*A
ATA =
        6375
>> ATb = A'*b
ATb =
        4595
>> ATA\ATb
ans =
    0.7208
>> c2 = ATA\ATb
c2 =
    0.7208
>> e2 = b-(A*c2)
e2 =
    5.3961
    2.7922
    3.1882
    3.5843
    3.9804
    2.3765
    0.1686
   -7.0392
>> diary off
```