18.06 Midterm Exam 1, Spring, 2001

Name ______________________ Optional Code ______________

Recitation Instructor ______________ Email Address ______________

Recitation Time ________________

This midterm is closed book and closed notes. No calculators, laptops, cell phones or other electronic devices may be used during the exam.

There are 3 problems. Good luck.

1. (20pts.) Find a general formula for the solutions of the following linear system of equations,

$$\begin{array}{rrrrcr} -x_1 & +3x_2 & & +2x_4 & = & 1 \\ 4x_2 & -12x_2 & +2x_3 & -4x_4 & = & -4 \\ -7x_1 & +21x_2 & +2x_3 & +18x_4 & = & 7 \end{array}$$

2. (40pts.) Let $A = \begin{pmatrix} 1 & 1 & b \\ a & b & b-a \\ 1 & 1 & 0 \end{pmatrix}$.

   (a) For $a = 2$ and $b = 1$, find the inverse of A.

   (b) For which values of a and b is the matrix A not invertible, i.e. it has less than three pivots?

   (c) For what values of a and b is the rank of A equal to 3 ? For what values is it equal to 2, equal to 1 ?

   (d) For $a = b = 2$, describe the nullspace of A.

3. (40pts.) Let $A = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$.

   (a) For what vectors $\mathbf{b} = (b_1, b_2, b_3)^T$ does the linear system $A\mathbf{x} = \mathbf{b}$ have a solution?

   (b) Prove that the column space of A is made up of those vectors $(x, y, z)^T \in \mathbb{R}^3$ that satisfy $x + y + z = 0$.

   (c) Prove that the vectors $(x, y, z)^T \in \mathbb{R}^3$ that satisfy $x + y + z = c$ form a subspace of $\mathbb{R}^3$ if and only if $c = 0$.