18.06 Midterm Exam 2, Spring, 2001

Name ______________________________ Optional Code ____________________

Recitation Instructor ____________________ Email Address ____________________

Recitation Time ______________________

This midterm is closed book and closed notes. No calculators, laptops, cell phones or other electronic devices may be used during the exam.

There are 3 problems. Good luck.

1. (40pts.) Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 1 & -1 \\ 9 & 5 & 1 \\ 9 & 8 & 7 \end{pmatrix}$$

   (a) Find the rank of A.

   (b) Find a basis for the row space of A, and find a basis for the nullspace of A. What is the dimension of the nullspace of A?

   (c) What can you say about the relation between the rank and the dimension of the nullspace of A?

   (d) Verify that all vectors in your basis of the nullspace are orthogonal to all vectors in your basis of the row space.

2. (30pts.) Let $a, b \in \mathbb{R}$, and let

$$A = \begin{pmatrix} 1 & 2 & 3 & a \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & b \end{pmatrix}.$$

   (a) What are the dimensions of the four subspaces associated with the matrix A? This will of course depend on the values of a and b, and you should distinguish all different cases.

   (b) For $a = b = 1$, give a basis for the column space of A. Is this also a basis for $\mathbb{R}^3$? Justify your answer.

3. (30pts.) An experiment at the seven times $t = -3, -2, -1, 0, 1, 2, 3$ yields the consistent result $b = 0$, except at the last time ($t = 3$), when we get $b = 28$. We want the best straight line $b = C + Dt$ to fit these seven data points by least squares.

   (a) Write down the equation $A\mathbf{x} = \mathbf{b}$ with unknowns C and D that would be solved if a straight line exactly fit the data.

(b) Use the method of least squares to find the best fit values for C and D.

(c) This problem is really that of projecting the vector $\mathbf{b} = (0,0,0,0,0,0,28)^T$ onto a certain subspace. Give a basis for that subspace, and give the projection $\mathbf{p}$ of $\mathbf{b}$ onto that subspace.