2.5.4

$$A = \begin{array}{c} \\ \text{edge} \end{array} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{pmatrix} \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} ; \quad A^T = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{pmatrix}$$

C

$$A^T A = \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \quad ; \det(A^T A) = 2(4-1) + 1(-2-1) - 1(1+2)$$
$$= 6 - 3 - 3 = 0 \quad \therefore \underline{\text{singular}}$$

<u>symmetric</u>

<u>Null space</u>: $N(A) = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ (as all incident matrices)

$N(A^T) = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$ (one closed loop)

$\therefore \; N(A^T A) = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$; left & right the same, $(A^T A)^T = A^T A$

Now $A' = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$, $A'^T = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$; $A^T A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$

$\det(A^T A') = 4 - 1 = 3 \neq 0$; <u>nonsingular</u>

2.5.5 $C = \begin{pmatrix} c_1 & & 0 \\ & c_2 & \\ 0 & & c_3 \end{pmatrix}$; $A^T C A = \begin{pmatrix} c_1+c_3 & -c_1 & -c_3 \\ -c_1 & c_1+c_2 & -c_2 \\ -c_3 & -c_2 & c_2+c_3 \end{pmatrix}$

Again $\det \begin{pmatrix} c_1+c_3 & -c_1 \\ -c_1 & c_1+c_2 \end{pmatrix} = (c_1+c_2)(c_1+c_3) - c_1^2 = c_1c_2 + c_1c_3 + c_2c_3 \neq 0$

in general

$(c_1, c_2, c_3 \geq 0)$.

2.5.6.

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

<u>$N(A) = (1,1,1,1)^T$</u>

$N(A^T)$: loops

(a) (x_1, x_2, x_4) : $n_1 = (1, 0, 0, 1, -1, 0)^T$

(b) (x_4, x_2, x_3) : $n_2 = (0, 0, 1, -1, 0, 1)^T$

(c) (x_3, x_1, x_4) : $n_3 = (0, -1, 0, 0, 1, -1)^T$