

2.5.11

$$A' = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad A'^T = \begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

System: $\begin{cases} C^{-1}y + Ax = 0 \\ A^T y = f \end{cases} \Rightarrow \begin{pmatrix} 1 & 1/2 & 1/2 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} + \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1/2 & 1/2 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 6 \end{pmatrix}$$

Now, $y = -CAx = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = -\begin{pmatrix} 1 & -2 \\ -2 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = -\begin{pmatrix} -x_1 + x_2 \\ 2(-x_1 + x_2) \\ 2(x_1) \\ -x_3 \end{pmatrix}$

and $A^T CA x = -f \Rightarrow \begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} -x_1 + x_2 \\ -2x_1 + 2x_3 \\ 2x_2 \\ -x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -6 \end{pmatrix}$

$$\Rightarrow \begin{cases} -(-x_1 + x_2) - (-2x_1 + 2x_3) = -1 \\ -(-x_1 + x_2) + 2x_2 = -1 \\ (-2x_1 + 2x_3) - (-x_3) = -6 \end{cases} \Rightarrow \begin{cases} 3x_1 - x_2 - 2x_3 = -1 \\ -x_1 + 3x_2 = -1 \\ -2x_1 + 3x_3 = -6 \end{cases}$$

$$\begin{pmatrix} 3 & -1 & -2 \\ -1 & 3 & 0 \\ -2 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ -6 \end{pmatrix}$$

$$\downarrow L_1 = \begin{pmatrix} 1 & 1/3 & 1 \\ 2/3 & 0 & 1 \end{pmatrix} \quad \text{''} \quad \begin{pmatrix} 1 & 0 & 1/4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 & -2 & 1 & -1 \\ 0 & 8/3 & -2/3 & 1 & -4/3 \\ 0 & -2/3 & 5/3 & 1 & -20/3 \end{pmatrix} \xrightarrow{L_2} \begin{pmatrix} 3 & -1 & -2 & 1 & -1 \\ 0 & 8/3 & -2/3 & 1 & -4/3 \\ 0 & 0 & 3/2 & 1 & -27/2 \end{pmatrix}$$