

2.5.16 For a complete graph how many edges are there?  
 The graph has  $n$  nodes, all nodes connect to each other by a single edge, no self-loops.

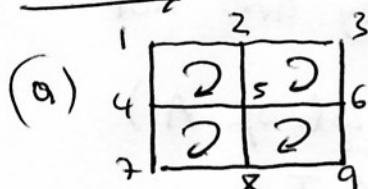
◀ This is the same as counting how many handshakes it takes for  $n$  people to shake hands with each other:

The first node connects to the others with  $n-1$  edges  
 " 2<sup>nd</sup> " " " " " "  $n-2$  edges  
 (we don't count the edge connecting to 1)  
 " 3<sup>rd</sup> " " " " " "  $n-3$  edges  
 (not counting edges to nodes 1, 2)  
 ⋮

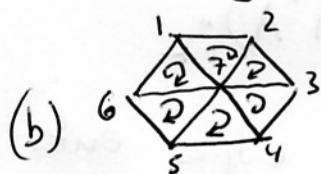
the  $(n-1)$ st , , , , , , , , , 1 edge

$$N = \sum_{i=1}^{n-1} i = \frac{n(n-1)}{2}$$

2.5.17 Verify Euler's formula  $(\# \text{ nodes}) - (\# \text{ edges}) + (\# \text{ loops}) = 1$



$$\left. \begin{array}{l} \text{Nodes: } 9 \\ \text{Edges: } 12 \\ \text{loops: } 4 \end{array} \right\} 9 - 12 + 4 = 1$$



$$\left. \begin{array}{l} \text{Nodes: } 7 \\ \text{edges: } 12 \\ \text{loops: } 6 \end{array} \right\} 7 - 12 + 6 = 1$$

2.5.18 Multiply matrices to find  $A^T A$  ( $A$  arbitrary incidence matrix) and guess how its entries come from the graph

(a) The diagonal of  $A^T A$  tells how many ? into each node

(b) The off diagonals  $-1$  or  $0$  tell which pairs of nodes are ?

~~reverse text~~