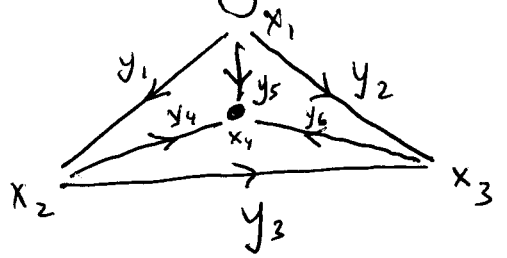
2.5.21 The adjacency matrix of a graph has $M_{ij} = 1$ if nodes i and j are connected by an edge (else $M_{ij} = 0$)

For the graph in figure with 4 nodes & 6 edges write down M and also M^2.

Why does $(M^2)_{ij}$ count the number of 2-step paths from node i to node j?



$$M = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \quad M^2 = \begin{pmatrix} 3 & 2 & 2 & 2 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 2 \\ 2 & 2 & 2 & 3 \end{pmatrix}$$

In general, $\sum_{j=1}^{n} M_{ij} M_{jk}$ has summands that are 1·1 if node i connects to node j & node k connects to node j (i.e. we can go from node i to node k via node j, i.e. in two steps) while they are zero if one of nodes i or k does not connect to node j. That is, the sum gives the number of 2-step paths from i to k.

(similarly the (i,k) entry of M^r gives the number of paths of length r from i to k).