

6. What 3 by 3 matrices represent the transformations that

- project every vector onto the x - y plane?
- reflect every vector through the x - y plane?
- rotate the x - y plane through 90° , leaving the z -axis alone?
- rotate the x - y plane, then x - z , then y - z , through 90° ?
- carry out the same three rotations, but each one through 180° ?

$$(c) \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Zeroes out
2-Component

inverts
2-component

 $R_2(\theta)$

(d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
 $R_x(90^\circ) \quad R_y(90^\circ) \quad R_z(90^\circ)$

$$(e) \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix} \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix}$$

 $R_x(90)$

7. On the space $\mathbf{P}_3$ of cubic polynomials, what matrix represents d^2/dt^2 ? Construct the 4 by 4 matrix from the standard basis $1, t, t^2, t^3$. Find its nullspace and column space. What do they mean in terms of polynomials?

polynomial $p_i(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \rightarrow \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$

$$\frac{d^3}{dt^3} p_4 = a_1 + 2a_2 t + 3a_3 t^2 \rightarrow \begin{pmatrix} a_1 \\ 2a_2 \\ 3a_3 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

Null space: $\text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\}$ (i.e. the constant polynomial)

Column space : $\text{span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$ (quadratic polys).

$$\frac{d^2}{dt^2} P_4 = 2a_2 + 6a_3 t \rightarrow \begin{pmatrix} 2a_2 \\ 6a_3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}$$

Null space: Linear polynomials $\left(\text{span}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}\right)$

Column space: linear polynomials

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