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Ex. 3.1.7. p. 142.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = U$$

$Ax = 0$ solve $Ux = 0$

x_1, x_3 - basic variables

x_2 — free variable

Let $x_2 = 1$ $\begin{cases} x_1 + 2 + x_3 = 0 \\ x_3 = 0 \end{cases}$ $\boxed{x = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}}$

$A^T y = 0$

$$A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 3 & 4 \end{pmatrix} \xrightarrow{P_{23}} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 4 & 6 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

y_1, y_2 – basic variables

y_3 – free variable

Let $y_3 = 1$ $\quad y_1 + 2y_2 + 3 = 0$

$\qquad\qquad y_2 + 1 = 0$ $\boxed{y = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}$

Ex. 3.1.9.

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} = U$$

$Ax = 0$

Solve: $Ux = 0$ $\qquad x_1, x_2$ — basic variables

x_3 – free variable

Let $x_3 = 1$ $\begin{cases} x_1 + x_2 + 2 = 0 \\ x_2 + 1 = 0 \end{cases}$ $x = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

The orthogonal complement is $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} c$, $\forall c \in R$.