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HW #5.

Ex. 3.1.16.
Find all vectors which are perpendicular to (1,4,4,1) and (2,9,8,2).

$$A = \begin{pmatrix} 1 & 4 & 4 & 1 \\ 2 & 9 & 8 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 4 & 4 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} = U$$

$Ax = 0 \Rightarrow Ux = 0$

$x_1, x_2$ - basic variables

$x_3, x_4$ - free variables

1) $x_3 = 1$, $x_4 = 0$ $\quad x_1 + 4x_2 + 4 = 0$, $x_2 = 0$ $\quad n_1 = \begin{pmatrix} -4 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

2) $x_3 = 0$, $x_4 = 1$ $\quad x_1 + 4x_2 + 1 = 0$, $x_2 = 0$ $\quad n_2 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

All these vectors are $\begin{pmatrix} -4 \\ 0 \\ 1 \\ 0 \end{pmatrix} c$ and $\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} c$, $\forall c \in \mathbb{R}$.

-1 rather $y = an_1 + bn_2$, $\forall a, b \in \mathbb{R}$

~~Ex 3.1.12~~. Find a basis for the nullspace of

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix} = U$$

$Ax = 0$

Let $x_3 = 1$ $\quad \begin{cases} x_1 + 2 = 0 \\ x_2 + 2 = 0 \end{cases}$ $\quad \boxed{x = \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}}$

Verify: $x^T A^T = \begin{pmatrix} -2 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \end{pmatrix}$

$x = x_r + x_n$

$Ax = Ax_r + Ax_n$, $Ax_n = 0$, $x_n$ is already found.

$$\begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} = \begin{pmatrix} x_{r1} \\ x_{r2} \\ x_{r3} \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix}}_{= x_r} + \underbrace{\begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}}_{= x_n}$$