

<II>(a) Find a basis for each of the four fundamental subspaces of the matrix

$$A := \begin{matrix} & \begin{matrix} 1 & 2 & 1 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ -1 \end{matrix} & \begin{matrix} 1 \\ -1 \\ 3 \end{matrix} & \begin{matrix} 2 \\ -5 \\ 9 \end{matrix} \end{matrix}. \quad \text{Now consider the system}$$

$A \underline{x} = \underline{b}$

$m=4$ $n=3$

(b) For (i) $\underline{b}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ and (ii) $\underline{b}_2 = \begin{pmatrix} 2 \\ -1 \\ 5 \\ -7 \end{pmatrix}$ check if the system is solvable without finding the solution.

(c) For $\underline{b}_1, \underline{b}_2$ in (b), if a solution exists, find all solutions.