Matrix Theory
Midterm Examination II

Fall 1990

Instructions: There are five (5) questions on this examination, worth a total of 100 points. Work all problems.

1. (20 points)
Solve the system:

$$\begin{array}{ccccccccc} c_0 & + & c_1 & + & c_2 & + & c_3 & = & 0 \\ c_0 & + & \mathrm{i}\, c_1 & - & c_2 & - & \mathrm{i}\, c_3 & = & 1 \\ c_0 & - & c_1 & + & c_2 & - & c_3 & = & 2 \\ c_0 & - & \mathrm{i}\, c_1 & - & c_2 & + & \mathrm{i} c_3 & = & 4\,. \end{array}$$

Give the inverse of the coefficient matrix W.

2. (a) (10 pts) If P is the matrix that projects $\mathcal{R}^n$ onto a subspace S, explain why every vector in S is an eigenvector and so is every vector in $S^{\perp}$. What are the eigenvalues?

(b) (10 pts) Find the eigenvalues and eigenvectors and the diagonalizing matrix S for

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \quad \textit{and} \quad B = \begin{bmatrix} 7 & 2 \\ -15 & -4 \end{bmatrix}$$

3. (20 points)
The following system has no solution:

$$Ax = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix} = b.$$

Sketch and solve a straight line fit that leads to the minimization of the quadratic $(C - D - 4)^2 + (C - 5)^2 + (C + D - 9)^2$. What is the projection of b onto the column space of A?

4. (a) (7 points) Find an orthonormal basis for the column space of

$$A = \begin{bmatrix} 1 & 3 & 4 & 5 & 7 \\ -6 & 6 & 8 & 0 & 8 \end{bmatrix}^T.$$

(b) (7 points) Write A as QR, where Q has orthonormal columns and R is upper triangular.

(c) (6 points) Find the least squares solution to $Ax = b$, if $b = (-3, 7, 1, 0, 4)^T$.

5. (a) (10 points) If A is nonsingular, show that there must be some permutation P for which PA has no zeroes on its main diagonal.

   (b) (10 points) Explain why the point (x, y) is on the line through $(2, 8)$ and $(4, 7)$ if

$$\det \begin{bmatrix} x & y & 1 \\ 2 & 8 & 1 \\ 4 & 7 & 1 \end{bmatrix} = 0, \quad \text{or} \quad x + 2y = 18.$$