p.427, App. B: 1 Find Jordan forms: Set 12

(a) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Column space: $\operatorname{span}\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$ → no intersection:

Null space $\operatorname{span}\left\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$

(i) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$: eigenvector, evalue 1.

(ii) No intersection, nothing to do

(iii) Null space outside of column space: $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = -\frac{1}{2}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$

i.e. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix}$ ↳ Jordan form

(b) $B = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$: $C(B) = \operatorname{span}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right\}$: $B\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$:

(i) no nontrivial eigenvector

(ii) intersection: $\operatorname{span}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right\}$; Need y: $By = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

(iii) Null space outside intersection: $n = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$: $B\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} = 0$

i.e. $B\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}}_{V} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}\underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\text{Jordan form}}$

$B = V\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}V^{-1}$