Math. 466

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Exam A

Due: in class, Tuesday, March 4

You may use results from the text, class notes & homework solutions. Be sure you justify precisely why a specific result is applicable to any given case. You must quote your sources precisely (i.e. eq. 7.82, p. 441, Arfken, or case #3, p. 9.3, class notes etc.).

Compute using contour integration

(1) $\displaystyle\oint_{|z|=1} \frac{1-\cos z}{(e^z-1)\sin z}\,dz$ (unit circle traversed counterclockwise)

(2) $\displaystyle\int_0^\infty \frac{dx}{x^2+2x+2}$

(3) $\displaystyle\int_{-\infty}^\infty \frac{\cos x}{\pi^2-4x^2}\,dx$

(4) Obtain the general term of a Taylor series expansion for the following functions around the indicated point. Where the function is multivalued give the results for all possible branches

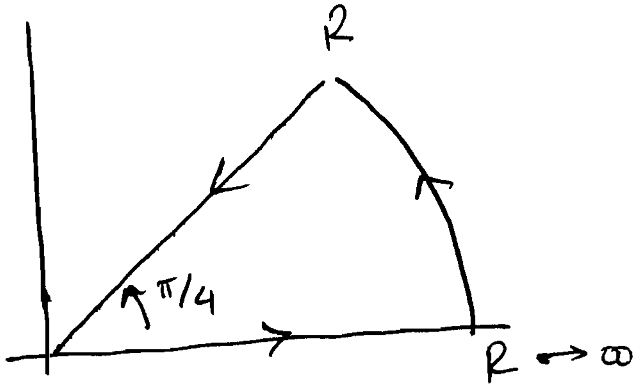
(a) $(1+z+z^2)^{-1}$ about $z_0=0$

(b) $z^{1/2}$ about $z_0=i$.

(5) Show that

$$\frac{1}{2}\sqrt{\frac{\pi}{2a}} = \int_0^{\infty} \cos(ax^2)\,dx = \int_0^{\infty} \sin(ax^2)\,dx$$

(Hint: use



Also, assume the formula

$$\int_0^{\infty} e^{-t^2}\,dt = \sqrt{\pi}/2$$).

You may take $a>0$.

(6) Use contour integration to show that

$$\int_0^{\pi} \cos^{2n}\theta\,d\theta = \pi\frac{(2n)!}{2^{2n}(n!)^2} = \pi\frac{(2n-1)!!}{(2n)!!}$$

$n=0,1,2,\dots$

(the notation $n!!$ is defined in text, sec. 10.1).

(7) It is known that $f(z)$ is analytic in the entire z-plane including ∞, except at the point $z = i/2$ where it has a simple pole and at $z = 2$ where it has a pole of order 2. In addition

$$\oint_{|z|=1} f(z)\,dz = 2\pi i, \qquad \oint_{|z|=3} f(z)\,dz = 0$$

and

$$\oint_{|z|=3} (z-2) f(z)\,dz = 0.$$

Find $f(z)$.