The reciprocal problem to problem I will now be formulated.

II: Minimize the expression

$$Q(\mathbf{q}, \mathbf{q}) - 2Q(\mathbf{q}, \mathbf{p}_0)$$

admitting for competition all vectors $\mathbf{q}$ for which $\mathbf{q} - \mathbf{q}_0$ is in Σ. If the minimum is attained for $\mathbf{q} = \mathbf{v}$, then by the same reasoning as above we can conclude that $\mathbf{v} - \mathbf{p}_0$ is in Ω.

Since $\mathbf{v} - \mathbf{q}_0$ is in Σ and $\mathbf{u} - \mathbf{q}_0$ is in Σ, the difference $\mathbf{u} - \mathbf{v}$ is in Σ; in the same way, from problem I we see that $\mathbf{v} - \mathbf{u}$ is in Ω. But since the two orthogonal subspaces have only the vector zero in common, it follows that $\mathbf{u} = \mathbf{v}$.

Thus problems I and II have the same solutions $\mathbf{u} = \mathbf{v}$.

By adding to the variational expressions in problems I and II the constant terms $Q(\mathbf{q}_0, \mathbf{q}_0)$ and $Q(\mathbf{p}_0, \mathbf{p}_0)$, respectively, we can state the problems in the following form:

I: Find the shortest distance from a fixed vector $\mathbf{q}_0$ to the linear set Ω_0, i.e. minimize

$$d(\mathbf{p}) = Q(\mathbf{p} - \mathbf{q}_0, \mathbf{p} - \mathbf{q}_0)$$

over all $\mathbf{p}$ in Ω_0.

II: Find the shortest distance from a fixed vector $\mathbf{p}_0$ to the linear set Σ_0, i.e. minimize

$$d(\mathbf{q}) = Q(\mathbf{q} - \mathbf{p}_0, \mathbf{q} - \mathbf{p}_0)$$

over all $\mathbf{q}$ in Σ_0.

Both minima d_1 and d_2 are attained by the same solution $\mathbf{p} = \mathbf{q} = \mathbf{u}$.

The reciprocal character of the two problems is expressed by the fact that the admissibility conditions of the one are the Euler conditions of the other.

Geometrically, the functions $\mathbf{p}$ and $\mathbf{q}$ are represented in Figure 2. A glance at this figure and the theorems of Pythagoras or Thales suggest, moreover, the relations

$$d_1 + d_2 = Q(\mathbf{p}_0 - \mathbf{q}_0, \mathbf{p}_0 - \mathbf{q}_0)$$

and

$$Q(\mathbf{u} - \mathbf{p}, \mathbf{u} - \mathbf{p}) + Q(\mathbf{u} - \mathbf{q}, \mathbf{u} - \mathbf{q}) = 4Q\left(\mathbf{u} - \frac{\mathbf{p} + \mathbf{q}}{2}, \mathbf{u} - \frac{\mathbf{p} + \mathbf{q}}{2}\right)$$