vectors belongs to Ω, whereas $\mathbf{p}$ itself is restricted to a linear set Ω_0 of vectors "parallel" to the linear subspace Ω.

As a first example, let the vector space Λ consist of vectors $\mathbf{p}$ which are pairs of functions defined in a region G; the norm is given by

$$Q(\mathbf{p}, \mathbf{p}) = \iint_G \mathbf{p}^2 \, dx \, dy$$

Take $\mathbf{p}_0 = \operatorname{grad} \varphi_0$ where φ_0 is a given function, and take as Ω the subspace of vectors $\boldsymbol{\omega} = \operatorname{grad} \varphi$ with $\varphi = 0$ on the boundary Γ of G. The subspace Σ, the orthogonal complement of Ω, consists of those vectors $\boldsymbol{\sigma}$ for which $\operatorname{div} \boldsymbol{\sigma} = 0$. If we choose $\mathbf{q}_0 = 0$, then for this example problem I is simply Dirichlet's problem: minimize $\iint_G (\varphi_x^2 + \varphi_y^2) \, dx \, dy$ subject to the condition $\varphi - \varphi_0 = 0$ on Γ.

Next, let the vector space Λ consist of functions p defined in a region G; the norm is given by

$$Q(p, p) = \iint_G p^2 \, dx \, dy.$$

Take $p_0 = \Delta\varphi_0$ where φ_0 is a given function, and take as Ω the subspace of functions $\omega = \Delta\varphi$ with φ and its normal derivative vanishing on Γ. The subspace Σ then consists of those functions σ for which $\Delta\sigma = 0$. If we choose $q_0 = 0$, then we have for φ the variational problem associated with the clamped plate.

Suppose that $\mathbf{u}$ minimizes the functional of problem I. Since $\mathbf{p} = \mathbf{u} + \epsilon\boldsymbol{\omega}$ (for any number ϵ and any element $\boldsymbol{\omega}$ in Ω) is also admissible, the vanishing of the first variation amounts to the condition

$$Q(\mathbf{u} - \mathbf{q}_0, \boldsymbol{\omega}) = 0 \qquad \text{for all } \boldsymbol{\omega} \text{ in } \Omega.$$

Hence the equivalent of Euler's equation and the accompanying natural boundary conditions for problem I is that $\mathbf{u} - \mathbf{q}_0$ is an element of Σ, the orthogonal complement of Ω.

The uniqueness of the minimizing vector $\mathbf{u}$ is now apparent. For, if $\mathbf{u}'$ also furnishes a minimum, then, as we have just seen, $\mathbf{u}' - \mathbf{q}_0$ is in Σ; consequently $(\mathbf{u} - \mathbf{q}_0) - (\mathbf{u}' - \mathbf{q}_0) = \mathbf{u} - \mathbf{u}'$ is a vector of Σ. But, in view of the admissibility conditions, $\mathbf{u}$ and $\mathbf{u}'$ are in Ω_0 so that their difference $\mathbf{u} - \mathbf{u}'$ lies in Ω. Since the subspaces Ω and Σ contain only the vector zero in their intersection, we see that $\mathbf{u}' = \mathbf{u}$.