where $\mathbf{p}$, $\mathbf{q}$ are any two vectors admissible respectively in the two reciprocal problems. These relations follow analytically from the fact that $\mathbf{u} - \mathbf{p}$ is in Ω, $\mathbf{u} - \mathbf{q}$ in Σ; hence $\mathbf{u} - \mathbf{p}$ and $\mathbf{u} - \mathbf{q}$ are orthogonal and

$$
\begin{aligned}
4Q\left(\mathbf{u} - \frac{\mathbf{p}+\mathbf{q}}{2},\quad \mathbf{u} - \frac{\mathbf{p}+\mathbf{q}}{2}\right) \\
&= Q(\mathbf{u} - \mathbf{p} + \mathbf{u} - \mathbf{q}, \mathbf{u} - \mathbf{p} + \mathbf{u} - \mathbf{q}) \\
&= Q(\mathbf{u} - \mathbf{p}, \mathbf{u} - \mathbf{p}) + Q(\mathbf{u} - \mathbf{q}, \mathbf{u} - \mathbf{q}) \\
&= Q(\mathbf{p} - \mathbf{q}, \mathbf{p} - \mathbf{q}).
\end{aligned}
$$

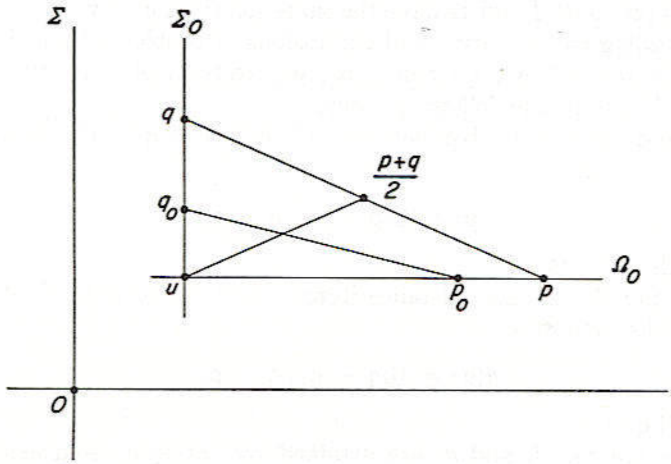

Figure 2

These considerations imply a remarkable fact: an upper bound for one of the two variational problems furnishes at the same time a lower bound for the reciprocal problem, since the sum of the minima is a priori known. This remark will prove relevant in connection with direct methods of the variational calculus.

Furthermore, the second relation,[1] corresponding to the theorem of Thales, states: The solution $\mathbf{u}$ common to both problems deviates from the arithmetical mean of two arbitrary functions $\mathbf{p}$, $\mathbf{q}$ admissible, respectively, in the two reciprocal problems by an amount equal to half the distance between the two functions $\mathbf{p}$ and $\mathbf{q}$. Hence, once

[1] In the present connection this relation was pointed out by Synge and Prager.