Chapter 9

Helical Navier-Stokes

The helical Navier-Stokes equations arise from imposing helical symmetry on the velocity field

$$\vec{u}(r,\theta,z,t) = \vec{u}(r,\phi,t) \tag{9.1}$$

and the axial pressure

$$p\hat{e}_z = [P_0(z) + q(r,\phi,t)\hat{e}_z] \tag{9.2}$$

where $P_0 = -Qz$ and

$$\phi = \theta + \alpha z. \tag{9.3}$$

A linear axial pressure gradient is the dominant component of the pressure field. Three partial differential equations arise from this helical symmetry:

$$-\triangle^*\psi = \xi + 2\alpha\gamma^4 u_0 \tag{9.4}$$

$$\frac{\partial u_0}{\partial t} = \frac{\nu}{\gamma^2}\triangle^* u_0 + J(\psi, u_0) + Q + 2\nu\alpha\xi \tag{9.5}$$

$$\frac{\partial \xi}{\partial t} = \nu\triangle^*\Big(\frac{\xi}{\gamma^2}\Big) - J(\xi,\psi) +$$

$$2\alpha\gamma^4[J(u_0,\psi) + \alpha u_0\frac{\partial u_0}{\partial \phi}] - 4\nu\alpha^2\gamma^4\xi - 2\nu\alpha\gamma^2\triangle^* u_0 \tag{9.6}$$

where

$$\triangle^* f = \frac{1}{r}\frac{\partial}{\partial r}\Big(r\gamma^2\frac{\partial f}{\partial r}\Big) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \phi^2} \tag{9.7}$$